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MEASURE OF ELECTRIC REACTIVE POWER

Generated electric power can either be dissipated or occur in the form of an electric field and a magnetic field. By replacing the voltage in the well-known equation for dissipated power (energy) with its derivative (a function orthogonal to the voltage) the average electric and magnetic field power exchanged between an electrical object and the rest of the closed electric power system was obtained. The measure of the (reactive) power is the surface area of the closed loop, which the object's characteristic forms in current-voltage coordinates.

Keywords: electric power, reactive power

1. INTRODUCTION

Not all generated electric power results in work performance and heat generation. It is the average electric power, which converts into work and heat. And it is unclear what other actions electric power converts to. Besides active power, apparent power and reactive power as well as a formal relationship between active power, reactive power and apparent power, i.e. a power triangle, have been defined in order to investigate electric power phenomena. The power triangle is unquestionable if the object is linear and the voltage is sinusoidal. If the voltage is not sinusoidal, the problem emerges: how to define reactive power in such a way that power triangle condition is satisfied. But this difficulty has not led to the rejection of the power triangle for nonsinusoidal waveforms. On the contrary, new orthogonal apparent power components suitable for the description of linear and nonlinear objects are sought [4, 5, 12, 14, 16]. But the additional components, as a rule, are not additive and they are not suitable for arbitrary objects.

An in-depth analysis of the electric power problem is made in this paper. It is shown that a definition of electric power stems from the equations of electrodynamics, assuming that electric power is transmitted only by a current flowing in conductors. Moreover, a general concept of an electrical object and basic phenomena which occur in it are defined.

The electrical phenomena occurring in the object can be roughly modelled by electric circuits. An electric circuit can also be a model object. Hence the properties of the characteristics of a model electric circuit in current-voltage and some other coordinates linearly dependent on the current-voltage coordinates were investigated. The results of the investigations show that the circuit's characteristics in current-voltage coordinates represent correctly electric and magnetic field power conversion and the surface enclosed by a line in the coordinates is a unique measure of reactive power. The investigations were carried out through simulation for assumed voltage waveforms and properties of electric circuit elements. The results were verified experimentally using real objects [11].

2. ELECTRIC POWER

It follows from the equations of electrodynamics (the Maxwell equations)

$$\operatorname{rot}\mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t},$$
$$\operatorname{rot}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \qquad (1)$$

that the Poynting vector flux

$$\mathbf{\Pi} = \mathbf{E} \times \mathbf{H}, \qquad (2)$$

penetrating a closed surface is equal to the total power of the electromagnetic field in volume V of the space confined by the surface:

$$p = -\oint \mathbf{\Pi} d\mathbf{s} = \int_V \mathbf{EJ}\, dv + \int_V \left(\mathbf{E}\frac{\partial \mathbf{D}}{\partial t} + \mathbf{H}\frac{\partial \mathbf{B}}{\partial t} \right) dv. \qquad (3)$$

Total power p consists of power, which converts into heat

$$p_s = \int_V \mathbf{EJ}\, dv \qquad (4)$$

and electric and magnetic field power

$$p_d = \int_V \left(\mathbf{E}\frac{\partial \mathbf{D}}{\partial t} + \mathbf{H}\frac{\partial \mathbf{B}}{\partial t} \right) dv. \qquad (5)$$

In Eqs. (1 - 5), **E** is an electric field intensity vector, **H -** a magnetic field intensity vector, **J -** a current density vector, **D** - an electric induction vector, **B** - a magnetic induction vector.

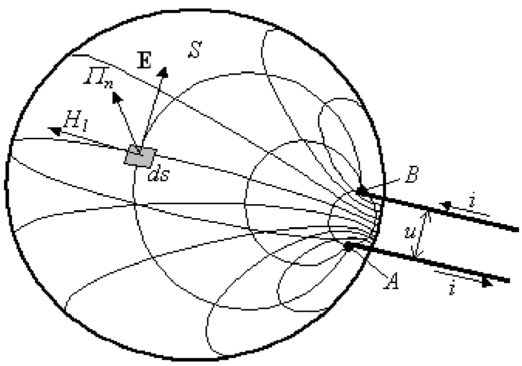


Fig. 1. Arrangement of vectors of electric field intensity **E**, tangent component of magnetic field H_l and normal component Poynting vector Π_n on closed surface surrounding electric object.

From Eqs. (2) and (3) the total power of an electric field in a space confined by closed surface S (Fig. 1), to which the electric field intensity vector **E** is tangent, can be calculated using the absolute value E of the vector and magnetic field strength component H_l tangent to surface S and perpendicular to vector **E**. This is possible because the normal component of the Poynting vector is equal to

$$\Pi_n = EH_l \qquad (6)$$

and the Poynting vector flux is expressed as follows

$$p = -\oint \Pi_n ds = -\int_A^B E\left(\oint H_l dl\right) dr . \tag{7}$$

In electric power engineering it is assumed that electric power is transmitted only by the flow of electric charge in conductors since dielectric displacement current

$$i_D = \frac{d\phi_e}{dt} \tag{8}$$

is negligibly small at power frequencies. In Eq. (8), ϕ_e stands for an electric induction flux. Electric charge may flow through a confined surface and the entire flow can be enclosed by an integration line. Then the circulation in Eq. (7) is equal to current i in the conductor (Fig.1)

$$\oint H_l dl = i . \tag{9}$$

regardless of the coordinates on closed surface S, and Eq. (7) is reduced to

$$p = -ui , \tag{10}$$

where

$$u = \int_A^B E dr \tag{11}$$

is the voltage between points A and B. The negative sign in Eq. (10) has a formal meaning and depends on whether power is supplied to or received from an object. If power is supplied, a positive sign is used.

In electromagnetic field power Eq. (3) there is a special term (4) for power converted into heat. The power can be considered as static (other than zero even for constant electric and magnetic fields) and expressed as

$$p_s = ui_s , \tag{12}$$

where i_s - the current flowing through supplementary resistance uniquely dependent on voltage.

Term (5) in Eq. (3) defines dynamic power which occurs only in variable and rotational fields. Since the dielectric shift current is omitted in the power systems the term can be expressed as

$$p_d = i_d \frac{d\psi}{dt} , \tag{13}$$

where i_d - current producing magnetic flux ψ.

Time-varying and closed-loop magnetic flux ψ induces a rotational electric field with such field intensity **E** that

$$\oint \mathbf{E}\, d\mathbf{r} = e = -\frac{d\psi}{dt}\,. \tag{14}$$

Voltage e causes the a current in electric power system circuits, which, in turn, generates flux ψ since in voltage period T the rotational magnetic field is bound to be converted into a rotational electric field and then the latter is bound to be converted into a rotational magnetic field. If the system is in a state of equilibrium, after period T the value of flux ψ within volume V will be repeated.

The equation of electric power (supplied to an object), written on the basis of relations (12), (13) and (14), has this form

$$p(t) = ui = ui_s - ei_d\,. \tag{15}$$

Power $p(t)$ is an additive quantity. If it is supplied to an object via many conductors, the total power is equal to the sum of the power supplied by the particular conductors.

The variable electric and magnetic fields in the space confined by closed surface S are not potential. The voltage between points A and B (Fig. 1) depends on the position of the line connecting the two points. The same value of voltage (11) is obtained only for lines tangent to the electric field intensity vector with a constant absolute value. If, however, it is assumed that the electric field is potential, surface S can be reduced to a surface embracing only the conductors' terminals and the object within the surface can be replaced by a simple (abstract) electric circuit.

3. ELECTRICAL OBJECT

In general, an electrical object can be considered as a space confined by a closed surface to/from which electric power is supplied/received by conductors carrying an electric current (Fig. 2).

The sum of instantaneous current values in all the conductors crossing the closed surface is obviously equal to zero

$$\sum_{k=1}^{N_1} i_k = 0\,. \tag{16}$$

Voltage u_k is the difference between the potential of the k-th conductor and that of the common point. The common point's potential can be, in principal, arbitrary - usually it is the earth's potential.

The total instantaneous power transmitted by all the conductors is equal to

$$p(t) = \sum_{k=1}^{N_1} u_k i_k\,. \tag{17}$$

In principle, only two electric power processes occur in the object:

1. electric power is dissipated (ultimately converted to heat) or generated at the expanse of nonelectric (usually thermal) power,
2. electric power in the object occurs in the form of an electric field and a magnetic field.

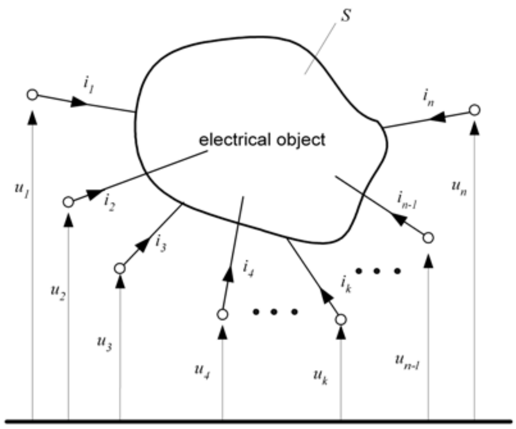


Fig. 2. Electrical object.

In order to investigate the phenomena, the object is replaced by a system of electric circuits connected to the individual conductors and to the common point. The resistance in the equivalent circuit models electric power dissipation, the inductance models power in the form of a magnetic field and capacitance models power in the form of an electric field. If electric power is generated in the object, the power sources are modelled by the voltage sources in the equivalent circuits. Basic electric power phenomena can be modelled separately by simple equivalent circuits in which the parameters: resistance, inductance and capacitance are connected in parallel or in series.

4. ELECTRIC POWER PARAMETERS

The power supplied to the object under the influence of a constant voltage in a steady state (when the current does not change over time) fully converts into work and heat. The power of an alternating current has no such property. Under the influence of a variable voltage a quite high alternating current may flow and the performed work and the released heat will be close to zero since the power of an alternating current may cause the mutual conversion of the electric field and the magnetic field (reactive interaction).

It is the average electric power which is converted in period T of a voltage or current waveform (active power):

$$P = \frac{1}{T}\int_{t_k}^{T+t_k} p\,dt = \frac{1}{T}\int_{t_k}^{T+t_k} ui\,dt\,. \tag{18}$$

For any voltage and current waveform the following relation holds:

$$P \le UI\,, \tag{19}$$

where: U - rms voltage, I - rms current. Product

$$S = UI \tag{20}$$

defines apparent power. Apparent power is the highest active power value for any waveforms of given rms voltage and current, which occurs when the current waveform is proportional to the voltage waveform. Parameter S is not additive (does not satisfy the balance condition).

Power factor

$$PF = \frac{P}{S} \tag{21}$$

defines the degree of use of the electric power supplied to the object by a current in one conductor.

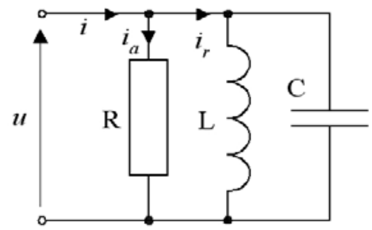


Fig. 3. Electric circuit used in investigations.

If the object is linear, parameters: resistance *R*, inductance *L*, and capacitance *C* of the equivalent circuit are constant and the sinusoidal voltage applied to the circuit

$$u = U_m \sin(\omega t) \tag{22}$$

produces sinusoidal current

$$i = I_m \sin(\omega t - \varphi). \tag{23}$$

Then the active power is equal to

$$P = UI\cos\varphi = S\cos\varphi \tag{24}$$

and for a given rms voltage it depends solely on the circuit resistance (Fig. 3)

$$P = \frac{U^2}{R}. \tag{25}$$

Reactive power for sinusoidal voltage and current waveforms

$$Q = UI\sin\varphi, \tag{26}$$

was defined so that

$$S^2 = P^2 + Q^2. \tag{27}$$

Equation (27) describes the power triangle (Fig. 4) which has become a fundamental canon in electrotechnics. Power *Q* for a given rms voltage and a given frequency depends solely on the circuit's inductance and capacitance (Fig. 3)

$$Q = U^2\left(\frac{1}{\omega L} - \omega C\right). \tag{28}$$

Thus it is a measure of the power phenomena resulting from the changes in the electric and magnetic fields in the circuit.

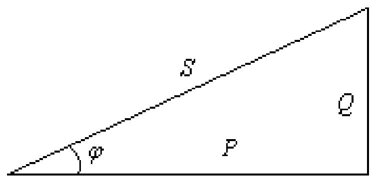


Fig. 4 Power triangle.

For deformed waveforms reactive power can be expressed as a sum of the active powers of appropriate current and voltage harmonics.

$$P = U_0 I_0 + \sum_{n=1}^{\infty} U_n I_n \cos\varphi_n . \tag{29}$$

By analogy with (21), (23) and (26) C. I. Budeanu [2] defined reactive power for deformed waveforms:

$$Q_B = \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n . \tag{30}$$

Because relation (30) is a linear combination of relation (29), reactive power Q_B (similarly as active power P) is additive. Moreover, reactive power Q_B is orthogonal to active power P. But the power triangle condition (27) is not met since it follows from the Cauchy inequality that

$$S^2 \geq P^2 + Q_B{}^2 . \tag{31}$$

Therefore C. I. Budeanu introduced an additional apparent power component orthogonal to active power P and reactive power Q_B, referred to as deformation power (symbol D). In this way a power cuboid (Fig. 5), in which

$$S^2 = P^2 + Q_B{}^2 + D^2 , \tag{32}$$

was created. There are cases when deformation power D is unrelated to the deformation of current relative to voltage [3]. It has also been found that in not all cases the compensation of reactive power Q_B improves the power factor [3]. Nonetheless C. I. Budeanu's definition of power has gained recognition and it is used.

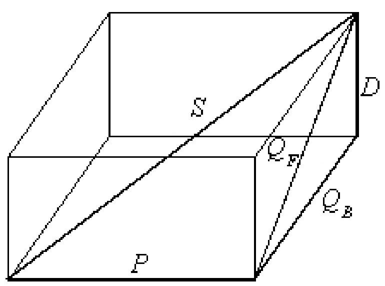


Fig. 5. Power cuboid.

If the circuit's resistance R, inductance L and capacitance C (Fig. 3) are constant, then current i_r flowing through reactance elements L and C is orthogonal to current i_a flowing through resistance R. S. Fryze exploited this property to define the reactive power of deformed currents [9].

Since

$$i = i_a + i_r \tag{33}$$

and currents i_a and i_r are orthogonal, their rms values satisfy this equation

$$I^2 = I_a^{\,2} + I_r^{\,2}. \tag{34}$$

If the above equation is multiplied by the square of the rms voltage applied to the circuit, one gets

$$S^2 = P^2 + Q_F^{\,2}. \tag{35}$$

Thus the power triangle condition for the linear circuit (Fig. 3) and any supply voltage waveform is fulfilled (Fig. 5). But reactive power Q_F defined on the basis of Eq. (35) is not additive, which means that it does not apply to arbitrary objects.

The search for proper orthogonal components of the current drawn by objects (including nonlinear ones) has been pursued for over 70 years now. A great number of works on the subject have been published (some of them are reviewed in [4, 5, 8]). But the reactive power problem has not been solved. This means that, in general, the current flowing in a conductor between an object and the rest of the electric power system cannot be separated into orthogonal components.

The electric energy transmitted by the current in one conductor and converted into work and heat in one conversion cycle is defined by this equation

$$\int_{t_k}^{t_k+T} ui\,dt = \int_{t_k}^{t_k+T} ui_s\,dt - \int_{t_k}^{t_k+T} ei_d\,dt \tag{36}$$

derived from Eq. (15). Since generally two energy processes occur in an object, one of which is described by equation (36), the other process can be described by a similar equation with the voltages and currents replaced with functions orthogonal to them. By replacing voltages u and e by their derivatives one gets

$$A_0 = \oint i\,du = \oint i_s\,du - \oint i_d\,de. \tag{37}$$

Geometrically, the equation (37) represents the surface area of the loops which the object characteristic forms in current-voltage (i, u) coordinates [1].

If quantity A_0 is a measure of reactive power, then the term

$$A_{0s} = \oint i_s\,du \tag{38}$$

should be equal to zero since power $p_s = ui_s$ (12) entirely converts into heat. Term (38) is actually equal to zero, but only when the current is a function of the voltage

$$i_s = i_s(u). \tag{39}$$

Relation (39) must be unique and so an inverse function

$$u = u(i_s) \tag{40}$$

must exist.

Quantity A_1, obtained by replacing voltages u, and e in Eq. (36) with their integrals (and so also with their orthogonal functions) has properties similar to those of A_0

$$A_1 = \oint \psi_e dq = \oint \psi_e dq_s - \oint \psi dq_d \tag{41}$$

The integrals represent respectively: equivalent flux ψ_e, and actual flux ψ which occurs in the object. Symbol q in Eq. (41) stands for an electric charge.

Any number of quantities with properties characteristic of reactive power can be theoretically determined [11, 21]:

$$A_m = \oint D^k u\, d(D^l i), \tag{42}$$

where D^k denotes k-time differentiation if k is a positive integer or k-time integration if k is a negative integer; numbers k and l must satisfy the condition: $k + l \rightarrow$ an even number.

All quantities A_m are additive because they are obtained through the linear transformation of Eq. (36).

The idea of defining reactive power by means of integrals containing quantities orthogonal to current or voltage was put forward as early as in the 1920s. M. Iliovici [10] formulated two definitions of reactive power:

$$Q_I = -\frac{1}{2\pi}\oint ide\,, \tag{43}$$

$$Q'_I = -\frac{2\pi}{T^2}\oint \psi dq\,. \tag{44}$$

Circulations in formulas (43) and (44) are equal to A_0 and A_1, respectively, assuming that component $p_s = ui_s$ does not result in reactive power. Integral definitions of power are used sometimes [6, 17, 18, 20, 23], but they have not gained wide recognition since they bear no relation to apparent power and the power triangle [11, 22].

A measure of reactive power was selected from the above quantities (having properties characteristic of reactive power) by testing. An object in the form of a circuit (Fig. 3) was used for the tests. Such an object (with assumed parameters) can be easily realized and its properties are determined easily.

5. CHARACTERISTICS IN COORDINATES i, u AND q, ψ

5.1. Linear circuit

In a linear circuit, sinusoidal voltage (22) produces sinusoidal current (23). Functions (22) and (23) are parametric equations of an ellipse in coordinates i, u. The surface area of the ellipse is

$$A_0 = 2\pi\, UI \sin\varphi\,, \tag{45}$$

where U, I - rms voltage and current.

Also the equivalent magnetic flux

$$\psi = \int u dt = -\frac{U_m}{\omega}\cos\omega t \tag{46}$$

and the electric charge

$$q = \int i dt = -\frac{I_m}{\omega}\cos(\omega t - \varphi) \tag{47}$$

form, in coordinates q, ψ, an ellipse with surface area

$$A_1 = \frac{T^2}{2\pi} UI \sin\varphi . \tag{48}$$

Surface areas A_0 and A_1 are proportional to the reactive power for sinusoidal waveforms

$$Q = \frac{1}{2\pi} A_0 = \frac{2\pi}{T^2} A_1 . \tag{49}$$

Thus for reactive power Q_0 defined on the basis of quantity A_0 one should adopt a normalizing coefficient $1/2\pi$, and for reactive power Q_1 defined on the basis of quantity A_1 - a normalizing coefficient $2\pi/T^2$:

$$Q_0 = \frac{1}{2\pi} A_0 , \tag{50}$$

$$Q_1 = \frac{2\pi}{T^2} A_1 . \tag{51}$$

If the voltage waveform is not sinusoidal, the values of Q_0 and Q_1 will not be equal. This means that at least one of them is false.

Voltage u (having any waveform) applied to the circuit (Fig. 3) produces current i defined by this equation

$$i = \frac{u}{R} + \frac{1}{L}\int u dt + C\frac{du}{dt} . \tag{52}$$

Powers Q_0 and Q_1 calculated from the above equation are

$$Q_0 = \frac{T}{2\pi}\left(\frac{1}{L}U^2 - C\dot{U}^2\right), \tag{53}$$

$$Q_1 = \frac{2\pi}{T}\left(\frac{1}{L}\Psi^2 - CU^2\right), \tag{54}$$

where $\dot{U}$ - an rms derivative voltage, Ψ - an rms equivalent flux (a voltage integral). For a given supply voltage powers Q_0 and Q_1 depend on the equivalent circuit's capacitance and inductance, i.e. on the elements modelling the occurrence of an electric field and a magnetic field in the object.

From the system of Eqs. (53) and (54) one can calculate the optimum capacitance (or inductance) which, when connected in parallel to a circuit (also a nonlinear one), results in the minimization of the rms current drawn by the circuit. The optimum capacitance is obtained from condition $Q_0 = 0$ and the optimum inductance from condition $Q_1 = 0$. This optimum is well known and used [12, 14, 15, 16].

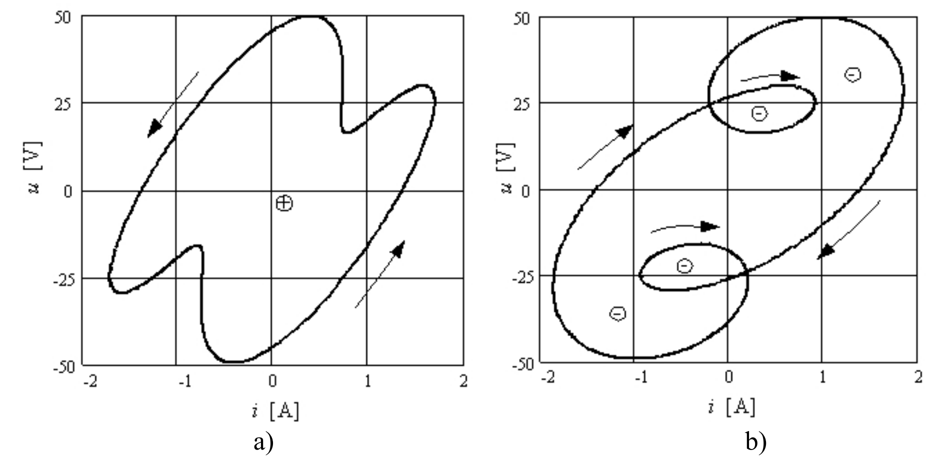


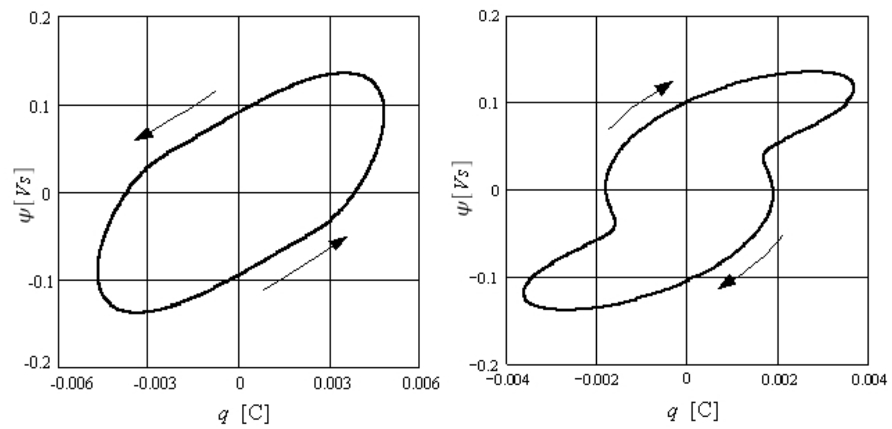
Fig. 6. Characteristics in coordinates u, i: a) circuit RL, b) circuit RC.

If only resistance and inductance occur in the electric circuit, the magnetic field power in the voltage waveform period in a state of power equilibrium must completely convert into electric field power outside the circuit and return to the circuit in the form of a magnetic field. Then the characteristic in coordinates i, u is a closed line running around all the partial surfaces in the same direction (anticlockwise). An example of such a characteristic when the voltage supplying the circuit contains only the first harmonic and the third harmonic:

$$u = U_{1m} \sin\omega t + U_{3m} \sin(3\omega t - \varphi_3), \tag{55}$$

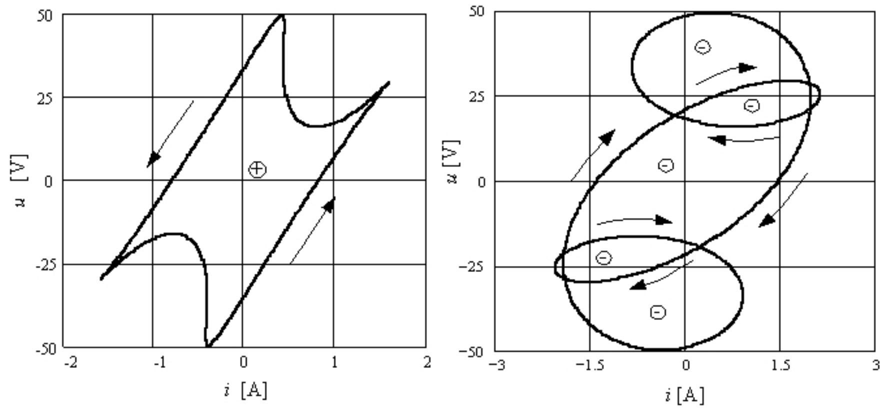
is shown in figure 6a. The electric field power in a circuit with only resistance and capacitance behaves in a similar way. The characteristic for such a system supplied with voltage (55) is shown in Fig. 6b (clockwise run). The characteristics in coordinates q, ψ (Fig. 7) have similar properties as those of the characteristics in coordinates i, u.

Only magnetic field power (Fig. 8a) or only electric field power (Fig. 8b) or both magnetic field power and electric field (Fig. 8c) can be exchanged between a circuit having a resistance, an inductance and a capacitance and the remaining part of a closed electric power system. By changing the circuit's capacitance or inductance a state can be reached when reactive power $Q_0 = 0$ or $Q_1 = 0$ (53, 54). The shape of the characteristic shown in Fig. 8c shows that the neutralization of reactive power Q_0 or Q_1 generally does not mean that electric field power and magnetic field power are not exchanged between the circuit and the rest of the system.

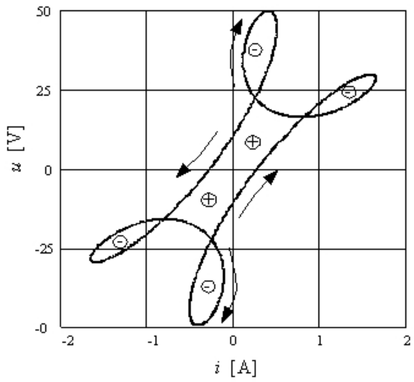


a) b)

Fig. 7. Characteristics in coordinates ψ,q for: a) circuit *RL*, b) circuit *RC*.



a) b)



c)

Fig. 8. Characteristics of circuit *RLC* in coordinates u,i: a) when inductive power predominates, b) when capacitive power predominates, c) when capacitive power and inductive power are comparable.

If the electric circuit has only a resistance and the resistance is constant (does not change over time), electric field power and magnetic field power definitely are not exchanged between the circuit and the rest of the system. The two characteristics: in coordinates i, u and q, ψ are reduced to line segments (Fig. 9). Powers Q_0 and Q_1 have the same zero values, i.e. they give a correct response.

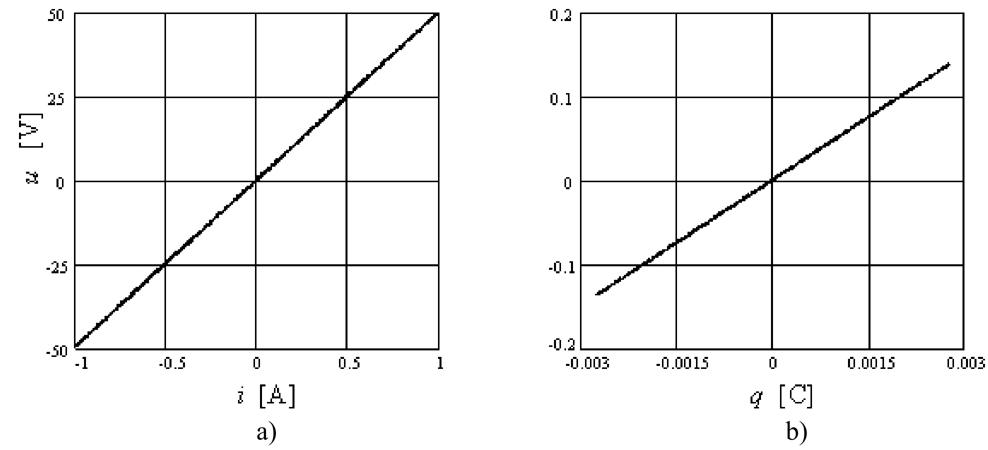


Fig. 9. Characteristics of circuit which includes constant resistance: a) in coordinates *u,i* ; b) in coordinates ψ,q.

5.2. Nonlinear circuit

Neither will electric field power and magnetic field power be exchanged between the circuit and the rest of the system if the resistance is not constant and it is the only element of the circuit.

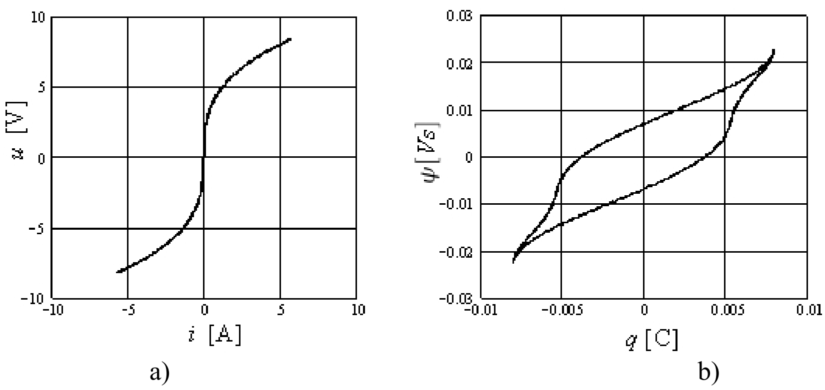
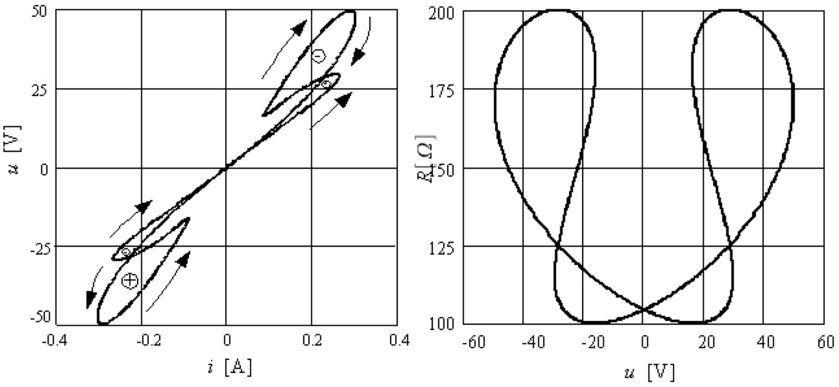


Fig. 10. Characteristics of a system which includes nonlinear resistance: a) in coordinates *u, i*; b) in coordinates ψ,q.

A characteristic in coordinates i, u and a characteristic in coordinates q, ψ for a circuit which has only a resistance, but not a constant one, is shown in Fig. 10. The circuit's resistance depends explicitly on the voltage in accordance with this equation

$$\frac{1}{R}=\frac{1}{R_0}\left[1+\left(\frac{u}{U_{1m}}\right)^2\right], \tag{56}$$

and the voltage changes according to relation (55) in which $U_{3m} = 0.5U_{1m}$ and $\varphi_3 = \pi/3$ are adopted. The characteristic in coordinates i, u is a line segment, whereas the characteristic in coordinates q, ψ forms a loop encompassing the surface other than zero. Hence parameter Q_0 has a zero value and it correctly indicates the absence of reactive power, whereas parameter Q_1 is not equal to zero, which means that it give false information. This property of the characteristic in coordinates q, ψ excludes parameter Q_1 as a measure of reactive power. Also reactive powers Q_B and Q_F are not always equal to zero when the characteristic in coordinates i, u reduces itself to a line segment. The reduction of the characteristic in coordinates i, u to a line segment when there are no reactive elements in the circuit has been verified for different unique resistance-voltage relations and different deformed voltage waveforms. It has also been verified that the characteristics in other coordinates $D^l i$, $D^k u$, where $k+l$ - an even integer, generally do not reduce themselves to a line segment when there is no reactive power [11, 20].



a) b)

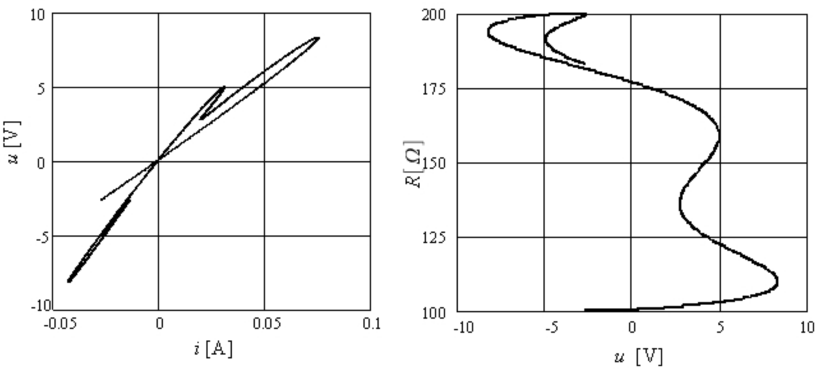
Fig. 11: a) characteristic of a circuit which includes only resistance equivocally dependent on voltage, but varying periodically; b) voltage-resistance relation.

When it is static, the circuit's resistance depends explicitly on voltage. If it varies over time irrespective of voltage, but periodically at the voltage frequency, for example in accordance with this equation

$$R = R_0\left(1+\sin^2\omega t\right), \tag{57}$$

then its dependence on voltage is equivocal and it has the form of a closed loop (Fig. 11b). The characteristic in coordinates i, u also has the form of a closed loop (Fig. 11a), which follows from assumption (57).

In order for the resistance to change independently of voltage, energy which does not come directly from the voltage source is needed. This energy disturbs the state of equilibrium, causing additional oscillations, and consequently the exchange of electric field power and magnetic field power. In actual objects the equivocal equivalent resistance-voltage dependence also occurs when the variable and rotational electric and magnetic fields perform work or cause the dissipation of electric power into heat. Such objects cannot be modelled solely by means of resistance.



a) b)

Fig. 12 a) characteristic in coordinates u,i for circuit which includes only resistance equivocally dependent on voltage; b) resistance-voltage relation.

If resistance changes over time nonsynchronously or nonperiodically with voltage (Fig. 13b), the characteristic in coordinates *i*, *u* does not form a closed loop (Fig. 13a) and it cannot be uniquely assigned the parameter Q_0, i.e. reactive power defined for periodic waveforms.

6. CONCLUSION

The exchange of electric field power and magnetic field power between an object and the rest of the electric power system is represented by the object's characteristic in coordinates (*i*, *u*). In the period of the voltage waveform the characteristic in coordinates (*i*, *u*) forms a closed loop. The direction in which the loop runs when electric field power is exchanged is opposite to the direction in which the loop runs when magnetic field power is exchanged. If both electric field power and magnetic field power are exchanged between the object and the rest of the system, then some of the surface area within the loop is encircled in one direction while the other is encircled in the other direction.

The surface area in coordinates *i*, *u* is a unique measure of reactive power defined as the average electric and magnetic field power exchanged in the voltage waveform period between the object and the rest of the electric power system.

A zero reactive power value is a necessary, but not sufficient, indicator of the compensation of the electric and magnetic field power flow. If the flows of electric field power and magnetic field power compensate, then not only the reactive power is equal to zero, but also the object's characteristic in coordinates *i*, *u* is reduced to a line segment.

REFERENCES

1. Bajorek J., Kolasa J., Kosobudzki G., Nowak J.: *Geometrical Interpretation of Power of Nonsinusoidal Currents*, Conference Proceedings, Elektrotechnika Prądów Niesinusoidalnych - EPN2000, Zielona Góra 2000 (in Polish).
2. Budeanu C.I.: *Reactive and Fictitious Powers*, Romanian National Institute, Bucharest, Pub. no. 2, 1927.
3. Czarnecki L.S.: *Powers and Compensation in Circuits with Periodic Voltages and Currents, Part 1: Budeanu's Power Theory: 60 Years of Illusions* , Jakość i Użytkowanie Energii Elektrycznej, 1997, vol. 3, no. 1, pp. 37 - 43, 1997 (in Polish).
4. Czarnecki L.S.: *Powers and Compensation in Circuits with Periodic Voltages and Currents, Part 2: Outline of the History of Power Theory Development*, Jakość i Użytkowanie Energii Elektrycznej, 1997, vol. 3, no. 2, pp. 37 - 46 (in Polish).
5. Emanuel A.E.: *Power in Nonsinusoidal Situations, A Review of Definitions and Physical Meaning*, IEEE Transactions on Power Delivery, vol. 5, no. 3, 1990.
6. Erlicki M., Emanuel–Eigeles A.: *New Aspects of Power Factor Improvement*, IEEE Transactions on Industry and General App., vol. IGA - 4, no. 4, July/August 1968.
7. Filipski P. S.: *Apparent Power – a Misleading Quantity in the Non-Sinusoidal Power Theory: Are all Non–Sinusoidal Power Theories Doomed to Fail?* ETEP vol. 3, no. 1, 1993, pp. 21 - 26.
8. Filipski P.S., Baghzoul Y., Cox M.D.: *Discussion of Power Definitions Contained in the IEEE Dictionary*, IEEE Transactions on Power Delivery, vol. 9, no. 3, 1994.
9. Fryze S.: *Active, Reactive and Apparent Powers in Networks with Nonsinusoidal Waveforms of Voltage and Current*, Przegląd Elektrotechniczny, no. 7, 8, 1931 (in Polish).
10. Iliovici M.: *the definition and measurement reactive power and energy*, Bull. Soc. Franc. Electriciens, 1925 vol. 5, pp. 931 - 956. (in French)
11. Kosobudzki G.: *Reactive power - simulation and measurement* Reports of the Institute of Electrical Machines, Drives and Metrology of the Wroclaw University of Technology. 2004 Ser. PRE no. 5 - PhD dissertation (in Polish).
12. Kusters N.L., Moore W. J. M.: *On the Definition of Reactive Power under Non-Sinusoidal Conditions*, IEEE Transactions on Power Apparatus and Systems, vol. PAS - 99, no. 5, 1980.
13. Sharon D.: *Power Factor Definitions and Power Transfer Quality in Nonsinusoidal Situation*. IEEE Transactions on Instrumentation and Measurement, vol. 45, no. 3, 1996.
14. Shepherd W., Zakikhani P.: *Suggested definition and power factor improvement in nonlinear systems*, Proc

IEE 1972 119, 1361 - 1362.
15. Shepherd W., Zakikhani P.: *Power-factor compensation of thyristor-controlled single-phase load*. PROC. IEE, 1973, vol. 120, no. 2, pp. 245 - 246.
16. Shepherd W., Zakikhani P.: *Reactive-Power Definitions and Power-Factor Improvement in Nonlinear Systems*. PROC. IEE, 1973, vol. 120, no. 6, pp. 704 - 706.
17. Superti Furga G., Pinola L.: *The Mean Generalized Content: A Conservative Quantity in Periodically-Forced Non-Linear Networks*. ETEP 1994 vol. 4, no. 3, pp. 205 - 211.
18. Superti Furga G.: *Searching for a Generalization of the Reactive Power - a Proposal*, ETEP 1994 vol. 4, no. 5, pp. 411 - 417.
19. Willems J. L., Ferrero A.: Is there a relationship between Non-Active Currents and Fluctuations in the Transmitted Power? ETEP 1998 Vol. 8, No. 4, pp. 265 - 270.
20. Fodor G., Tevan G.: *Powers and Compensation in Networks in Periodic State*, Archiv fur Elektrotechnik 65, pp. 3 - 40, 1982.
21. Kosobudzki G.: *Integrated Method of Calculating Power In Single-Phase Circuit With Non-Sinusoidal Current And Voltage*, Scientific Papers of the Institute of Electrical Machines, Drives and Metrology of the Wroclaw University of Technology, no. 53, Wrocław 2003, pp. 375 - 382 (in Polish).
22. Kolasa J., Kosobudzki G.: *Relations between powers of linear receiver supplied nonsinusoidal voltage*, Conference Proceedings, Kongres Metrologii KM2004, Wrocław 2004 (in Polish).
23. Majewskij O., A. , Goncarov P. Ju. Defining reactive power of electric non-linear circuits, Elektricestvo. no. 3/1967, pp. 22 - 27 (in Russian).

MIARA ELEKTRYCZNEJ MOCY BIERNEJ

Streszczenie

Z równań elektrodynamiki wynika, że występują tylko dwa zasadnicze procesy elektroenergetyczne: energia elektryczna może być rozpraszana oraz może być gromadzona w postaci pola elektrycznego i magnetycznego. Proces rozpraszania energii elektrycznej można opisać za pomocą mocy czynnej, której miarą może być powierzchnia zamkniętej pętli, jaką tworzy charakterystyka obiektu we współrzędnych prąd elektryczny – strumień magnetyczny. Dla opisu właściwości energetycznych obiektu stosuje się, oprócz mocy czynnej, również inne parametry mocy elektrycznej: moc pozorną oraz moc bierną. Moc pozorna jest największą wartością mocy czynnej możliwą do osiągnięcia dla danej wartości skutecznej napięcia. Moc bierna jest jednoznacznie zdefiniowana dla sinusoidalnych przebiegów napięcia i prądu. Dla przebiegów niesinusoidalnych moc bierną definiuje się najczęściej przy założeniu, że prąd w przewodach systemu elektroenergetycznego da się rozłożyć na składowe ortogonalne. Badania prawdziwości tego założenia prowadzone są od ponad 70 lat.

Jeśli moc bierna ma opisywać proces wymiany energii pola elektrycznego i magnetycznego, to jej miarę można określić podobnie jak miarę mocy czynnej. Ponieważ w obiekcie zachodzą tylko dwa podstawowe procesy elektroenergetyczne, z których jeden jest opisany przez moc czynną, to drugi można opisać zamieniając w równaniu definiującym moc czynną, na przykład napięcie, na pochodną napięcia względem czasu, czyli na funkcję ortogonalną do napięcia. Miarą tak zdefiniowanej mocy biernej jest powierzchnia zamkniętej pętli, jaką tworzy charakterystyka obiektu we współrzędnych prąd – napięcie. Miara ta jest jednoznaczna. Charakterystyka obiektu w tych współrzędnych redukuje się do odcinka linii (nie tworzy pętli), gdy parametrem zastępczym obiektu jest tylko rezystancja jednoznacznie zależna od napięcia, czyli gdy między obiektem a resztą systemu elektroenergetycznego nie jest wymieniana energia pola elektrycznego i magnetycznego. Takich właściwości nie mają charakterystyki w innych współrzędnych. Również moc bierna definiowana według innych koncepcji nie zawsze jest równa zeru, gdy w obiekcie nie ma zmiennego w czasie pola elektrycznego i magnetycznego.

Jeśli zależność rezystancji zastępczej od napięcia nie jest jednoznaczna, to oznacza, że do obiektu jest dostarczana dodatkowa energia bezpośrednio nie pochodząca ze źródła napięcia, która zakłóca stan równowagi powodując dodatkowe oscylacje, a więc pojawienie się mocy biernej.